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SYSTEMS ANALYSIS

CYCLES IN ECONOMIC SYSTEMS WITH OPEN LABOR MARKETS

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A dynamic macromodel of an economic system with a monopsonic labor market and variable labor resources is considered. The initiation conditions for classical business cycles are investigated for this model. Bifurcation analysis of the model is performed to estimate the effect of the minimum wage on the way such cycles form. The results of theoretical analysis are supplemented with numerical experiments.

Keywords: *dynamic systems, bifurcation, monopsony, labor migration, business cycles.*

INTRODUCTION

Various types of imperfect competition with employer priority are typical of the labor markets of many postsocialist countries. This is due to the low activity of trade unions and their dependence on employers, the insufficient development of small and medium business, imperfect national legislation, monofunctional cities, and the low mobility of the population. Strong financial and industrial groups, which intensively formed at the beginning of the 21st century in many countries appeared after the dissolution of the Soviet Union, also promote imperfect competition. Note that it is a labor market with employer's monopolism (a monopsonic market) that allows gaining the maximal profit in a limited time. This promotes such an imperfect competition in an unstable socioeconomic and political situation typical of the early market reforms.

Monopsony due to financial and industrial groups that control monofunctional cities is manifested especially strongly in individual regions of postsocialist countries. For example, according to [1], 16 largest Russian corporations control (directly or indirectly) the industry of 53% of monofunctional cities. More than 60% of the population of some Russian regions lives in such cities [1]. The country's average value of this parameter is 26%. Geographical preconditions for monopsony are strong in Eastern Siberia (Krasnoyarsk and Norilsk industrial hubs), in the north of Western Siberia (areas of oil- and gas production,) and in the Ural region, where oligarchic financial and industrial structures are especially strong. Almost every third city in Ukraine (mainly small) and more than half of city-type settlements are monofunctional [2]. The greatest concentration of such settlements is observed in regions with advanced mining (first of all coal), chemical, and metallurgical industry, and with some sub-branches of food-processing industry (sugaring and alcohol production). The influence of financial and industrial groups is also significant for these branches.

Thus, it is reasonable to consider centralized business structures that supervise a significant part of the economy of large regions as monopolist employers. Note that despite the traditionally low mobility of the population in the former USSR, the labor market of such regions remains open. Noneconomic factors interfering population shifts within one country were weakened at the end of the 1990s due to the liberalization of political systems of postsocialist countries. There is also a transborder (mainly semilegal and illegal) labor migration among these countries. At the first sight, migration processes eliminate monopsony in the labor market; however, this is not exactly so. If the intensity of such processes is rather insignificant, and employers can neither predict migration flows nor manage them, they will perceive the available

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manpower as an exogenous parameter influencing the labor supply. As in the case of pure monopsony, wages will be determined by employers in view of the current values of the parameter specified. Then the labor market will remain monopsonic but will be open since decisions of employers influence the amount of manpower in subsequent instants of time.

An analysis of the influence of such a labor market on general economic dynamics is of interest.

Monopsony is known to generate a number of negative social and economic processes. Low wages, typical of this type of imperfect competition, limit the final demand for goods and services, which may cause economic recession [3]. However, unemployment in its classical meaning (excessive supply of labor) is also insignificant in the case of monopsony. An excessive (for the employer) manpower is superseded from the labor market due to the low remuneration. If such a market is open, preconditions are formed for a large-scale labor migration reducing the labor supply and decelerating the reduction of wages. This process will certainly affect both the depth of recession and the duration of economic depression. The remuneration of labor in a monopsonic market rises abruptly under economic revival; however, the inequality of income distribution also increases. Intensive labor immigration allows employers to decelerate the growth of the remuneration of labor and to earn additional income. However, this may cause serious social problems. One of the features of a closed monopsonic labor market is that it generates classical business cycles with stages of growth, boom, recession, depression, and revival of economy [4]. Of interest is to analyze the features of economic dynamics in systems with the same type of imperfect competition in the open labor market discussed earlier. In the present paper, we consider a dynamic macromodel intended for these purposes. Its analysis dwells on cyclic processes and their comparison with the cycles observed in the model presented in [4]. The theoretical analysis is supplemented with results of numerical experiments, especially important in cases where analytical methods cannot be used.

The paper consists of an introduction, four sections, and conclusions. In Sec. 1, we consider a dynamic macromodel of a system with open monopsonic labor market and its special case. A qualitative analysis of the model for this case is performed in Sec. 2. In Sec. 3, we use some aspects of the interaction of an open monopsonic market with a foreign market. The results of numerical experiments on the model proposed are presented and analyzed in Sec. 4. In the conclusions, we formulate conclusions and discuss the lines of further studies.

1. MODEL OF AN ECONOMIC SYSTEM WITH OPEN MONOPSONIC LABOR MARKET

Consider the following structure of a labor market. There are L employed persons and a unique employer being the exclusive consumer of manpower. Let the number of employees be too great, and the individual labor supply of each of them be too small to influence the remuneration of labor. The employees cannot coordinate their actions, and the employer knows the amount of the cumulative labor supply and demand for the production he manufactures using the manpower. The employer pursues his short-term interests, tending to maximize the income from the production. The model also assumes that the prices are invariant; therefore, the values of all the economic indicators will be nominal and real simultaneously. Assume that additional expenses associated with manpower (indirect wages-fund taxes, social deductions, etc.) are proportional to the wages fund W.

Let us consider a function $S(W, L)$ of the maximum amount of manpower that the employer can get with the given W and the given number L of employees. Actually, $S(W, L) = \max\limits_{\omega \in X(W)} \hat{L}_S(\omega, L)$, where $\hat{L}_S(\omega, L)$ is the function of the cumulative labor supply dependent on wage ω and on L; $X(W) = \{\omega : \omega \hat{L}_S(\omega, L) \le W, \omega \ge 0\}$. (If the minimum wage ω_0 is introduced, then $X(W) = \{\omega : \omega \hat{L}_S(\omega, L) \le W,\ \omega \ge \omega_0\}$. If the set $X(W)$ is empty, complete the definition $S(W, L) = 0$.) Note that the amount of manpower got by the employer for the wages fund he established is always maximum: a smaller amount of manpower allows manufacturing less product and earning smaller income for the same labor costs. The employer will reduce W if sales slowdown compels him to reduce the employment.

In what follows, we assume that the function $S(W, L)$ is nonnegative, continuous, and is zero for $W \le W_0$ ($W_0 = 0$ is possible in a special case). The quantity W_0 is determined by the minimum wage ω_0; in particular, if $\omega_0 = 0$, then $W_0 = 0$. Assume also that the function $S(W, L)$ is concave and monotonically increasing in W for each fixed L on the set $X_0 = \{W : W > W_0\}$, is twice continuously differentiable with respect to W on X_0, $S'_W(W, L) > 0$, and $S''_{WW}(W, L) < 0$. Note that if we discard the assumption of differentiability, the technical aspects of the analysis of the model become complicated but the qualitative conclusions mainly remain.

Similarly to [3], it is possible to show that under the assumptions made, the greatest income of the employer corresponds to the point of maximum of the function

$$F(W,L) = \min(lS(W,L), \alpha V) - (1+h)W, \tag{1}$$

with respect to the variable W, considered for a fixed L, where l is the labor productivity in terms of the value added, V is the demand for the product manufactured by the employer, α is the share of the value added in the price of this product, h is additional expenses for purchasing the manpower (indirect wages-fund taxes, social payments, etc.); let $H = h+1$. Since the function $F(W,L)$ is continuous and concave on the set X_0 with respect to W, its points of maximum can be the points where it is non-differentiable (with respect to W) or where its derivative with respect to W is equal to zero. The case where the point of maximum is W_0 can be excluded if we assume that the right-hand derivative $S_W^{+\prime}(W_0, L)$ with respect to W of the function $S(W,L)$ at the point $W = W_0$ is sufficiently large:

$$S_W^{+\prime}(W_0, L) > \frac{H}{l}$$

for all $L \geq 0$. Since the limiting labor supply in case of low remuneration is commonly rather great, the last assumption is quite adequate to the properties of actual labor markets.

Thus, the point of maximum, with respect to W, of the function $F(W,L)$ defined according to (1) and considered for fixed L is

$$W^* = \min\left(S_W^{-1}\left(\frac{\alpha V}{l}, L\right), (S'_W)^{-1}(Hl^{-1}, L)\right), \tag{2}$$

where $S_W^{-1}(X,L)$ is the function inverse to $S(W,L)$ with respect to the variable W (i.e., a solution of the equation $S(W,L) = X$ for given X and L), $(S'_W)^{-1}(\cdot)$ is the function inverse to $S'_W(W,L) = \dfrac{\partial S(W,L)}{\partial W}$ with respect to the same variable.

The model under study is dynamic, with continuously varying time t. Therefore, in what follows, we will consider the quantities V, W^*, and L as functions continuous in t. The number of employees L can take only integer values; however, it is possible to discard this requirement for a large L by choosing an appropriate scale and considering part-time employees (pupils, physically challenged persons, etc.).

The value of L in the model changes due to the migration of manpower. The driving force of the migration is the difference in the remuneration of labor in the domestic (within the framework of the system being modeled) and foreign labor markets. If we denote by $\overline{W}$ the wages fund in the foreign market scaled to the domestic market, then the intensity of migrations will be determined by the function $\overline{F}(W, \overline{W})$. And $\overline{F}(W, \overline{W}) > 0$ if $W > \overline{W}$ (i.e., a high remuneration of labor in the domestic market causes labor immigration); if $W < \overline{W}$, then $\overline{F}(W, \overline{W}) < 0$. For $W = \overline{W}$, we have $\overline{F}(W, \overline{W}) = 0$. By the assumptions, $\overline{F}(W, \overline{W})$ is a function continuously increasing in W and decreasing in $\overline{W}$ and satisfying the Lipschitz property in both variables. Changes in the variable L are defined by the differential equation

$$L' = \overline{F}(W^*, \overline{W}). \tag{3}$$

For simplicity, we assume in what follows that the quantity $\overline{W}$ and the function $\overline{F}(W, \overline{W})$ are not directly dependent on time t.

Changes in the demand V are determined by the differential equation

$$V' = a_1 W^* - a_2 V + a_3, \tag{4}$$

similar to that considered in [3]. Assume in (4) that $a_1 > 0$, $a_2 > 0$, and $a_3 \geq 0$ are some constants. The relationships (2)–(4) and the initial conditions

$$V(0) = V^0, \quad L(0) = L^0 \tag{5}$$

form the model under study.

The phase space of the system (2)–(4) is the set $E = \{(V, L): 0 \le V \le V_{\max},\ 0 \le L \le L_{\max}\}$, where $V_{\max}$ and $L_{\max}$ are some sufficiently large constants. In what follows, we will assume that a phase point achieves the boundary of E and then moves along it in a sliding mode [5] influenced by the vector field. Under the conditions considered later, the vector field of the system inside E is smooth, except for the switching line on which it remains continuous. Therefore, the solution $(L(t), V(t))$ of the problem (2)–(5) exists under the assumptions made earlier if $(V^0, L^0) \in E$. It is defined on the positive semiaxis and is a continuously differentiable function of t at all points except for those (if any) at which the solution $(L(t), V(t))$ either reaches the boundary of the set E or abandons it. The vector function $(L(t), V(t))$ is continuous at these points. The model (2)–(5) can also be considered under the assumption that the phase point stops once it reaches the boundary of E.

Then we analyze the system for a special case being of interest, where

$$S(W, L) = \begin{cases} L(W - W_0)^{1/2} & \text{if } W > W_0, \\ 0 & \text{if } W \le W_0. \end{cases}$$

$$\overline{F}(W, \overline{W}) = W - \overline{W},\ a_3 = 0.$$

Such functions satisfy the assumptions on the properties of $S(W, L)$ and $\overline{F}(W, \overline{W})$ made earlier. It is reasonable to assume that the basic trends in the dynamics of the system remain in the general case. It is easy to verify that the following inequalities are true of the system under study:

$$S'_W(W, L) = 0.5L(W - W_0)^{-1/2},\quad W > W_0,$$

$$S_W^{-1}\left(\frac{\alpha V}{l}, L\right) = \frac{\alpha^2 V^2}{l^2 L^2} + W_0, \tag{6}$$

$$(S'_W)^{-1}(Hl^{-1}, L) = \left(\frac{l}{2H}\right)^2 L^2 + W_0.$$

Of special interest is to analyze the dependence of the solutions of the system (2)–(4) on variations in W_0 for the given case. As we mentioned above, W_0 depends on the minimum wage ω_0, which is one of the major means of the governmental regulation of labor market under the conditions of imperfect competition [6]. We will perform such an analysis in the next section.

2. ANALYSIS OF THE MODEL

Note that the values of the constant terms on the right-hand sides of the differential equations do not usually influence the results of a qualitative analysis of the system. For simplicity, we assume these constants equal to unity and make appropriate changes of variables and thus rearrange the problem (2)–(5) in the following form, where $S_W^{-1}(\cdot)$ и $(S'_W)^{-1}(\cdot)$ are defined by (6):

$$L' = W - \overline{W}, \tag{7}$$

$$V' = aW - V, \tag{8}$$

$$W = \min\left(\delta + \gamma L^2, \delta + \frac{V^2}{L^2}\right), \tag{9}$$

$$V(0) = V^0,\quad L(0) = L^0, \tag{10}$$

where $\overline{W} > 0$, $a > 1$, $0 \le \delta \le \overline{W}$, and $\gamma > 0$ are some constants, $(V^0, L^0) \in E$.

We will first construct a switching line $\hat{H}$, which is the set of points such that the minimum in Eq. (9) is attained for both expressions simultaneously. As follows from the equation $\delta + \gamma L^2 = \delta + \frac{V^2}{L^2}$, $\hat{H}$ is a part of the parabola $V = \gamma^{1/2} L^2$, which belongs to E. The equalities $W = \delta + \gamma L^2$ and $W = \delta + \dfrac{V^2}{L^2}$ hold for points (V, L) over and under, respectively, the curve $\hat{H}$.

Let us determine the points at which the right-hand side of Eq. (7) becomes zero. Over the line $\hat{H}$, these points satisfy the condition $\delta + \gamma L^2 = \overline{W}$ or $L = \gamma^{-1/2}(\overline{W} - \delta)^{1/2}$. In what follows, we will denote the vertical line segment corresponding to this set by $\overline{H}_1$. Under the line $\hat{H}$, these points should satisfy the condition $\overline{W} = \delta + \dfrac{V^2}{L^2}$ or $V = L(\overline{W} - \delta)^{1/2}$. In what follows, we will denote the line segment corresponding to this set by $\overline{H}_2$.

Stationary points of the system (7)–(9) should satisfy the conditions $W = \overline{W}$ and $aW = V$; therefore, the equality $V = a\overline{W}$ holds at these points. Thus, the system (7)–(9) with the initial conditions (10) can have no more than one stationary point over the line $\hat{H}$ (the point of intersection of the horizontal line $V = a\overline{W}$ with the segment $\overline{H}_1$) and no more than one stationary point under $\hat{H}$ (the point of intersection of the above-mentioned horizontal line with the segment $\overline{H}_2$). Let us consider the case where both these points exist. The necessary and sufficient condition for them to exist is

$$a \geq \max(\gamma^{3/2}, \gamma^{-1/2})\left(1 - \frac{\delta}{\overline{W}}\right). \tag{11}$$

In what follows, by O_1 we denote the stationary point with the coordinates $(L^{(1)}, V^{(1)}) = (\gamma^{-1/2}(\overline{W} - \delta)^{1/2}, a\overline{W})$, overlying the $\hat{H}$, and by O_2 denote the point with the coordinates $(L^{(2)}, V^{(2)}) = (a\overline{W}(\overline{W} - \delta)^{1/2}, a\overline{W})$ under the $\hat{H}$. Determine the sets of points over and under, respectively, $\hat{H}$ for which the right-hand side of Eq. (8) becomes zero. The equality $V = a(\delta + \gamma L^2)$ holds for the points overlying $\hat{H}$. By $\widetilde{H}_1$ we denote a segment of this parabola belonging to E. As inequality (11) holds, this segment overlies $\hat{H}$. For the points under $\hat{H}$, the right-hand side of Eq. (8) is zero if the equality $V = a\left(\delta + \dfrac{V^2}{L^2}\right)$ holds. Hence

$$V = \frac{L^2}{2a} \pm \sqrt{\frac{L^4}{4a^2} - \delta L^2} \tag{12}$$

for $L \geq 2a\delta^{1/2}$.

The set of points satisfying Eq. (12) is formed by two branches. The first of them, $\widetilde{H}_2$, is a graph of a differentiable and increasing function that approaches the parabola $V = a^{-1}L^2 + a\delta$ from below as L increases without limit. The second branch, $\widetilde{H}_3$, is a graph of a differentiable and decreasing function of L that has the horizontal asymptote $V = a\delta$. The distance between $\widetilde{H}_3$ and this asymptote is of order $O\left(\dfrac{1}{L^2}\right)$ for sufficiently large L. For $\delta = 0$, the line $\widetilde{H}_3$ becomes the horizontal coordinate axis $V = 0$. The branches $\widetilde{H}_2$ and $\widetilde{H}_3$ intersect at the point B with the coordinates $(2a\delta^{1/2}, 2a\delta)$. For $\delta = 0$, the point B coincides with the origin of coordinates 0.

The right-hand side of Eq. (7) is positive at the points between the lines $\overline{H}_1$ and $\overline{H}_2$ and negative at all other points of the set E. The right-hand side of Eq. (8) is positive at the points between the curves $\widetilde{H}_1$ and $\widetilde{H}_2$ and at the points under the curve $\widetilde{H}_3$. At other points of the set E, it is negative.

Figure 1 shows the phase portrait of the system (7)–(9). By $D^{\gamma_1\gamma_2}$ $(\gamma_1, \gamma_2 = +,-)$ we denote the sets of points for which the right-hand side of Eq. (7) has the sign γ_1 and the right-hand side of Eq. (8) the sign γ_2. Let us consider these sets (domains) in detail.

The domain D^{--} is above the line $\widetilde{H}_1$, to the left of the vertical segment $\overline{H}_1$, and between the lines $\overline{H}_2$ and $\widetilde{H}_3$. The domain D^{+-} is located between $\overline{H}_1$ and $\widetilde{H}_1$, and also between $\widetilde{H}_2$ and $\overline{H}_2$. The domain D^{++} is bounded by the lines $\widetilde{H}_1$, $\overline{H}_1$, $\overline{H}_2$, and $\widetilde{H}_2$. The set D^{-+} is under the line $\widetilde{H}_1$ (to the left of $\overline{H}_1$) and under $\overline{H}_2$ and $\widetilde{H}_3$ (to the right of $\overline{H}_1$).

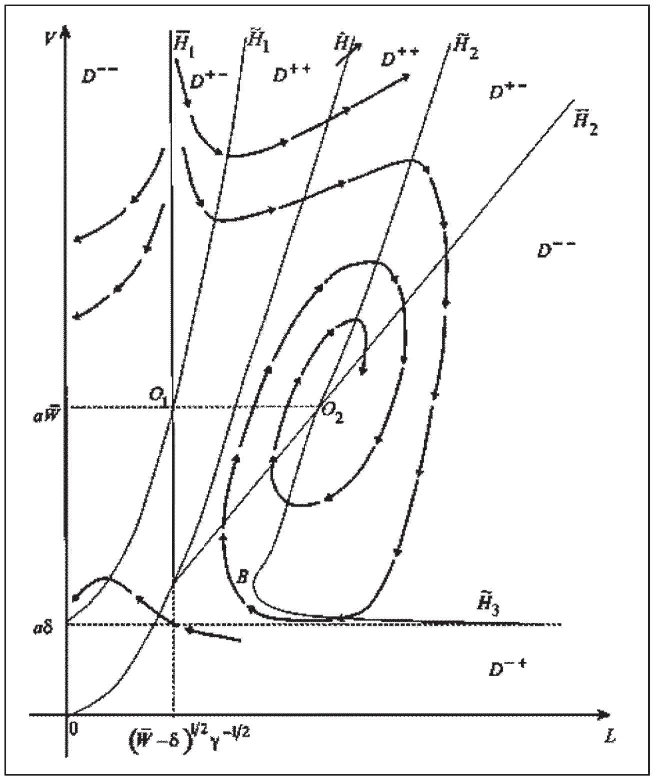

Fig. 1. Schematic phase portrait of the system, where $\rightarrow\rightarrow\rightarrow$ are phase trajectories.

Let us consider special features of the motion of the phase point on the above-mentioned sets. If the phase point is in the domain D^{--} to the left of the line $\overline{H}_1$, then its motion is accompanied by increase in the available manpower L and in the demand V for the product manufactured. Since the phase point is above the switching line $\hat{H}$,the wages fund $W = \delta + \gamma L^2$ decreases with L. This aggravates labor emigration and decreases the demand for the product manufactured. As a result, a feedback occurs: reduced remuneration of labor — reduced manpower because of intensified emigration — reduced remuneration resulted in a prolonged recession in production. This process causes the phase trajectory to reach the boundary of the space E at the point where $L = 0$, i.e., the dynamic system being modeled collapses because of a complete loss of manpower. Note that if the phase point appears near the line $\widetilde{H}_1$, then since the right-hand side of Eq. (8) is continuous, V' is close to zero, while L' substantially differs from zero. The phase point will continue moving to the axis $0V$ along the nearly-horizontal trajectory. Therefore, it could not intersect the line $\widetilde{H}_1$ and occur in the domain D^{-+} . Thus, if the phase point appears in the domain D^{--} to the left of $\overline{H}_1$, the economy will inevitably collapse because of prolonged recession of production.

The motion of the phase point in the domain D^{--} between the lines $\overline{H}_2$ and $\widetilde{H}_3$ will also be accompanied by both intensified labor emigration (decrease in L) and recession in production (decrease in V). As a result, the phase point will intersect either the curve $\widetilde{H}_2$ or the curve $\widetilde{H}_3$. In the latter case, the relationship $V = 0$ holds for $\delta = 0$ and economy collapses. For $\delta > 0$, the phase point intersected $\widetilde{H}_3$ will continue moving in the domain D^{-+} . The value of V' will be close to zero since the right-hand side of Eq. (8) is continuous and the phase trajectory will be a nearly horizontal line. Note that the curve $\widetilde{H}_3$ will also be a nearly horizontal straight line away from the point B, and phase trajectories that intersect this line at different points will be close to each other, i.e., they will stick together.

Consider now the motion of the phase point in the domain D^{-+} . The value of L will decrease because of continued emigration; however, V will start increasing due to the increase in the wages fund $W = \delta + \dfrac{V^2}{L^2}$. A lack of manpower lost during the economic recession compels the monopolist employer to increase wages in order to compensate for the decrease in the manpower supply. This increases the internal demand for consumer goods, which revives the economy. As wages in the domestic market exceed wages in the foreign market, labor emigration stops, the phase point intersects the line $\overline{H}_2$ and appears in the domain D^{++}. However, if the moving phase point appears to the left of the line $L = \gamma^{-1/2}(\overline{W} - \delta)^{1/2}$, it intersects $\widetilde{H}_1$ and appears in the domain D^{--}. In this case, the economy revival is temporary and is followed by recession, which results in collapse as $L = 0$. The motion of the phase point in the domain D^{++} is accompanied both by increase in manpower L due to immigration and by growth in demand V for the product manufactured. These variables increase especially fast at the points overlying the line $\hat{H}$, where the rate of their growth is proportional to L^2. A fast boom-like upturn of the economy corresponds to this. Two scenarios of further events are possible: (i) V and L continue growing up to the moment the phase point approaches the upper boundary of the phase space E (unless the equality $V = V_{\max}$ or $L = L_{\max}$ holds); (ii) the phase point moving along D^{++} intersects the line $\widetilde{H}_2$ and appears in the domain D^{+-}. The second scenario may be because of a considerable excess in the remuneration of labor in the domestic market as compared with the foreign one. Intensive labor immigration abruptly increases manpower supply, which allows the monopolist employer to substantially decrease the remuneration, which reduces the demand for the product, and the boom is followed by economic recession.

The motion of the phase point on the set D^{+-} is accompanied by reduced production. Over some time, the remuneration in the domestic market exceeds that in the foreign market. The continuing immigration leads to an excessive manpower, which, under conditions of monopsony, abruptly reduces the remuneration of labor and aggravates the recession. As the phase point intersects the line $\overline{H}_2$, the remuneration in the domestic and foreign markets coincide, immigration ceases, and the phase trajectory continues in the domain D^{--}. If the phase point moves between the lines $\overline{H}_1$ and $\widetilde{H}_1$, then the wages fund $W = \delta + \gamma L^2$ increases as the manpower L grows because of immigration. This revives the economy.

Thus, three types of the economy dynamics are possible for the system (7)–(9) for different initial conditions (10): (i) prolonged recession to the left of the line $\overline{H}_1$, which results in collapse of the economy; (ii) prolonged growth on the set D^{++}, after which the phase point reaches the upper boundary of the phase space E; and (iii) cyclic development around the stationary point O_2. In the latter case, economic cycles similar to a classical business cycle occur, with the stages of recession (motion of the phase point on the set D^{+-}), depression (motion of the point on D^{--}), revival (motion on D^{-+}), and growth followed by a boom (motion on D^{++}). By analogy with the model studied in [4] for a closed monopsonic labor market, an anaslysis of such cycles, in particular, bifurcation analysis, is of special interest. The bifurcation parameter is δ, whose value depends on the minimum wage. Since we will focus on cycles, we will restrict ourselves to the assumption that as the phase point reaches the boundary of E, it slides along it under the action of the vector field.

Let us classify the stationary points of the system.

The point O_1 and its vicinity are located over the curve $\hat{H}$. For points of this vicinity, $W = \delta + \gamma L^2$, and the Jacobian matrix J_1 for the system becomes

$$J_1 = \begin{pmatrix} 2\gamma L & 0 \\ 2a\gamma L & -1 \end{pmatrix}.$$

The value of this matrix at the point $O_1 = (\gamma^{-1/2}(\overline{W} - \delta)^{1/2}, a\overline{W})$, called the stability matrix, is defined by

$$A_1 = \begin{pmatrix} 2\gamma^{3/2}(\overline{W} - \delta)^{1/2} & 0 \\ 2a\gamma^{3/2}(\overline{W} - \delta)^{1/2} & -1 \end{pmatrix}.$$

The eigenvalues of the matrix A_1 are -1 and $2\gamma^{-1/2}(\overline{W} - \delta)^{1/2}$, they correspond to the eigenvectors $(0, 1)$ and $(1, 2\gamma^{-1/2}(\overline{W} - \delta)^{1/2})$. Thus, O_1 is a saddle point, and the separatrices entering O_1 belong to the vertical straight line $L = \gamma^{-1/2}(\overline{W} - \delta)^{1/2}$. The isoclinal line of horizontal inclinations of the system (7)–(9) in the domain above $\hat{H}$ is the parabola $V = a(\delta + \gamma L^2)$, i.e., the curve $\widetilde{H}_1$.

The phase point can reach O_1 by moving along the interface between the two above-mentioned scenarios of economic dynamics; O_1 is an unstable equilibrium point, any small variations in V and (or) L cause either inevitable recession or cyclic development. (Under the above assumptions on the possible sliding along the boundary of the phase space E, long-term growth becomes one of such cases of economic dynamics.)

The point O_2 and its vicinity underlie the curve $\hat{H}$. Similarly to the point O_1, the Jacobian matrix J_2 for the system and the stability matrix A_2 are defined by

$$J_2 = \begin{pmatrix} -2\dfrac{V^2}{L^3} & 2\dfrac{V}{L^2} \\ -2a\dfrac{V^2}{L^3} & 2a\dfrac{V}{L^2} - 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -2\dfrac{(\overline{W}-\delta)^{3/2}}{a\overline{W}} & \dfrac{\overline{W}-\delta}{a\overline{W}} \\ -2a\dfrac{(\overline{W}-\delta)^{3/2}}{\overline{W}} & \dfrac{\overline{W}-\delta}{\overline{W}} - 1 \end{pmatrix}.$$

In what follows, we will consider the case where the inequality (11) that ensures the existence of the point O_2 holds for any $\delta \ge 0$, whence

$$\gamma^{1/2} a > 1. \tag{13}$$

Note that since inequality (13) holds, the vector field of the system (7)–(9) is continuous on the line $\hat{H}$. Since

$$\mathrm{tr}\,(A_2) = 2(\overline{W}-\delta)\overline{W}^{-1}(1-(\overline{W}-\delta)a^{-1}) - 1,$$

$$\det\,(A_2) = 2(\overline{W}-\delta)^{3/2} a^{-1}\overline{W}^{-1} + 4(\overline{W}-\delta)^{5/2}\overline{W}^{-2}(1-a^{-1})$$

and the condition $a > 1$ is satisfied, $\det\,(A_2) > 0$. Assume that

$$(\mathrm{tr}\,(A_2))^2 - 4\det\,(A_2) < 0.$$

Then if $\mathrm{tr}\,(A_2) < 0$, then O_2 is a stable focal point of the system (7)–(9); if $\mathrm{tr}\,(A_2) > 0$, then O_2 is an unstable focal point of the system.

The isoclinal lines of the horizontal and vertical inclinations of the system for points under $\hat{H}$ are, respectively, $V = \left(\delta + \dfrac{V^2}{L^2}\right)$ and $V = L(\overline{W}-\delta)^{1/2}$, i.e., the curves $\tilde{H}_2$, $\tilde{H}_3$, and $\overline{H}_2$.

We will start the bifurcation analysis of the system for increasing δ from the case where $\mathrm{tr}\,(A_2) > 0$ and $\delta = 0$. Assume that the separatrix of the point O_1 emanating toward D^{++} belongs to the influence domain of a local invariant manifold [7] $\{(L,V): V = 0, 0 \le L < L_{\max}\}$. Figure 2 presents the phase portrait of the system for this case. As mentioned above, the curve $\tilde{H}_3$ coincides with the axis $0L$, which is why it is omitted in the figure.

Let us note the distinctive features of the motion of the phase point. As it passes through the set D^{+-}, between the lines $\tilde{H}_2$ and $\overline{H}_2$, the demand V rapidly (exponentially) decreases and the number of employees L slowly increases. If the phase point moving on the set D^{--} approaches the axis $0L$, then the approximate equalities $L' \approx -\overline{W}$ and $V' \approx 0$ hold. Thus, the phase trajectory will pass nearly parallel and close to the axis $0L$ and will finish at the origin of coordinates 0, which is a singular stable point of the system (7)–(9) considered under the curve $\hat{H}$.

Let the above-mentioned conditions be satisfied and the parameter $\delta > 0$ be small. Figure 3 represents the phase portrait of the system (7)–(9). Note that after intersecting the curve $\tilde{H}_3$, the phase point will continue its motion in the domain D^{-+} near this curve and almost parallel to it. Following [7], we conclude that there exists a locally exponentially stable invariant manifold $\mathfrak{I}_\delta = \{V = \Phi(L,\delta),\, 2a\delta \le L \le L_{\max}\}$ of the system, and $\Phi(L,\delta) \approx \dfrac{L^2}{2a} - \sqrt{\dfrac{L^4}{4a^2} - \delta L^2}$. Whence we may assume with a reasonable degree of accuracy that the point B of convergence of the curves $\tilde{H}_2$ and $\tilde{H}_3$ belongs to the above-mentioned manifold. Note that B is a cuspidal point, at which $L' = 2\delta - \overline{W}$ and $V' = 0$. Thus, for $\delta < \dfrac{1}{2}\overline{W}$, $L(t)$ decreases and $V(t)$ increases near this point, and after a time, the phase trajectory will intersect the switching line $\hat{H}$. The coordinates of the intersection point will be continuous functions of δ; the first coordinate will also be a monotonically increasing function. The separatrix emanating from the point O_1 into the domain D^{++} will intersect the line $\hat{H}$ for the given δ at the point $P(\delta)$. Repeating the previous reasoning, we conclude that the coordinates of $P(\delta)$ will be continuous functions, the first coordinate increasing with δ and vanishing for $\delta = 0$. Since the first coordinate of the point O_1 is a continuous monotonically decreasing function of δ, which vanishes for $\delta = \overline{W}$, there exists a δ^* such that the first coordinates of the points $P(\delta)$ and O_1 coincide for $\delta = \delta^*$ and a homoclinic loop Γ^* [8, 9] shown in Fig. 3 occurs in the system (7)–(9).

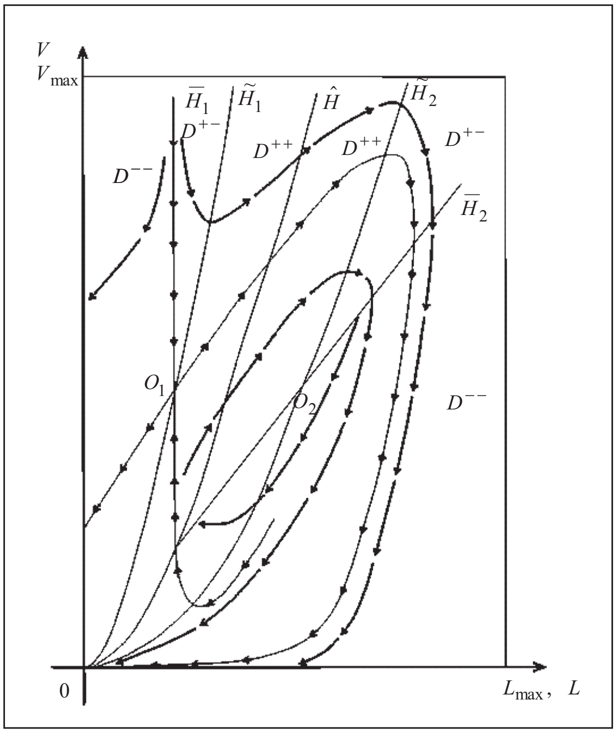

Fig. 2. Phase portrait of the system in the case where $\delta = 0$ and the point O_2 is unstable; $\to\to\to$ are phase trajectories, $\twoheadrightarrow\twoheadrightarrow$ are separatrices of the point O_1.

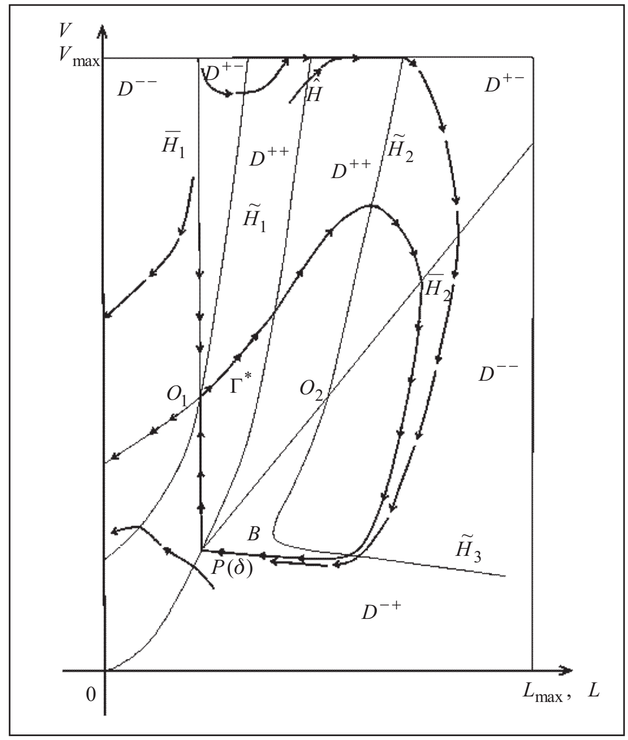

Fig. 3. Forming a stable homoclinic loop, where $\to\to\to$ are phase trajectories, $\twoheadrightarrow\twoheadrightarrow$ are separatrices of the point O_1 and sliding motion, $\longrightarrow$ is a homoclinic loop.

If the separatrix emanating from the point O_1 toward the set D^{++} completely belongs to the interior of the phase space E, then the stability of the homoclinic loop Γ^* will depend on the sign of the saddle value of the point O_1 [8, 9]. This value equals to $\dfrac{\partial W(V,L)}{\partial L} + \dfrac{\partial (aW(V,L) - V)}{\partial V}$, where the function $W(V,L)$ is given by (9). Note that the point O_1 and its vicinity are always located above the switching line $\hat{H}$; therefore, $W(V,L)$ is a differentiable function at the point O_1. As follows from [8, 9], a homoclinic loop is stable if the saddle value of the point O_1 is negative and is unstable if the saddle value is positive. As follows from the computations, the saddle value of the point O_1 is negative in our case.

When the emanating separatrix crosses the boundary of the space E, the phase point continues sliding along this boundary. The phase curves located in a sufficiently small vicinity of the separatrix will also cross the boundary of E at the points sufficiently close to the point of intersection of the boundary and the separatrix. Therefore, sliding along these curves will continu along the boundary. Note that the end point of the sliding will depend exclusively on the configuration of the vector field of the system, and the above-mentioned trajectories will coincide with the homoclinic curve after their separation from the boundary of E.

Let us consider the case where $\delta > \delta^*$. For a sufficiently small $\delta - \delta^*$, the first coordinate $P(\delta)$ will be greater than $\gamma^{-1/2}(\overline{W} - \delta)$. As follows from [8, 9], there is a unique stable limit cycle Γ_δ (Fig. 4) in the vicinity of Γ^*. The period of the motion of phase points along this cycle tends to the infinity as $\delta \to \delta^*$. There is a certain analogy with the dynamics of the system with a closed monopsonic labor market considered in [4], where a unique limit cycle was also generated when the bifurcation parameter increased.

As δ grows, the phase portrait of the system can vary in several ways. If Γ_δ is a unique limit cycle, it decreases as δ increases and, for some $\delta = \delta^{**}$, collapses at the point O_2, which becomes a stable focus with an extensive domain of attraction. Numerical experiments have confirmed that such a dynamics is possible. An increase in the minimum wage makes classical cycles in a system with open monopsonic market similar to postclassical cycles and thus reduces the duration of boom and depression.

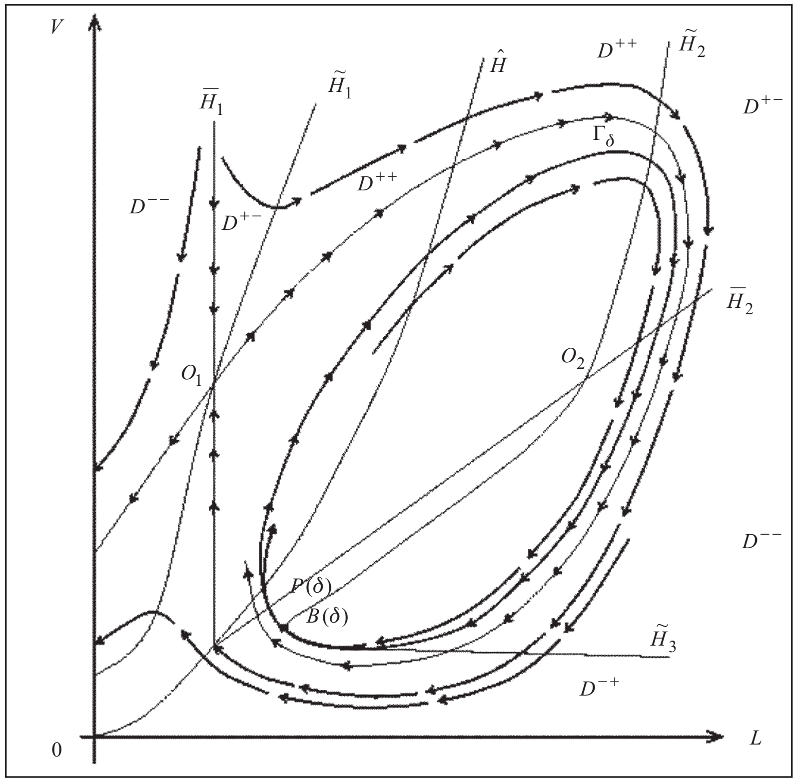

Fig. 4. Origin of a stable limit cycle, where $\rightarrow\rightarrow\rightarrow$ are phase trajectories, $\twoheadrightarrow\twoheadrightarrow$ are separatrices of the point O_1, $\longrightarrow$ is a stable limit cycle.

If the system has limit cycles other than Γ_δ or they appear as δ increases, there are also stable limit cycles among them. Then the system attains one of them depending on the initial conditions. There is also an analogy to the system having a closed monopsonic labor market [4].

Assume that the phase points of system (7)–(9) located on the above-mentioned separatrix do not belong to the domains of influence of the origin of coordinates and of the point O_2. Figure 5 schematically shows the phase portrait of such a system for $\delta = 0$. The point of intersection of the separatrix emanating from O_1 and passing through D^{++} with the switching line $\hat{H}$ is designated by $P(0)$.

Denote by $P(\delta)$ a similar point obtained with $\delta > 0$. As mentioned above, if δ increases, the point O_1 is shifted to the left and the point O_2 to the right along the $0L$axis, and the attraction of the point O_1 along the second coordinate axis and the attraction of the point O_2 intensify. Hence, the point $P(\delta)$ climbs along the curve $\hat{H}$; therefore, there exists a δ^* such that the homoclinic loop Γ^* originates for $\delta = \delta^*$. As mentioned above, the stability of the homoclinic loop depends on the sign of $\mathrm{tr}(A_1) = 2(\overline{W} - \delta)\gamma^{1/2} - 1$ for $\delta = \delta^*$. Since δ^* (dependent on a and γ) is usually small, we may approximately assume that the homoclinic loop is unstable if $2\overline{W}\gamma^{1/2} > 1$ and is stable if $2\overline{W}\gamma^{1/2} < 1$ (Fig. 6).

Let us consider in detail the case of unstable loop. If $\delta > \delta^*$ but $\delta - \delta^*$ is sufficiently small, a family of stable limit cycles Γ_δ exists in the vicinity of Γ^*. If O_2 is an unstable focal point, then the system dynamics is similar to that considered earlier. If O_2 is a stable focal point, then there is at least one unstable cycle inside Γ_δ. This cycle expands as δ increases, while Γ_δ contracts. Therefore, the cycles will merge at some value of δ, which will result in a semistable cycle. It will disappear with increase in the bifurcation parameter, and the system will have a stable focus with an extensive domain of attraction and a saddle point. Similar effects were also noticed for the model presented in [4].

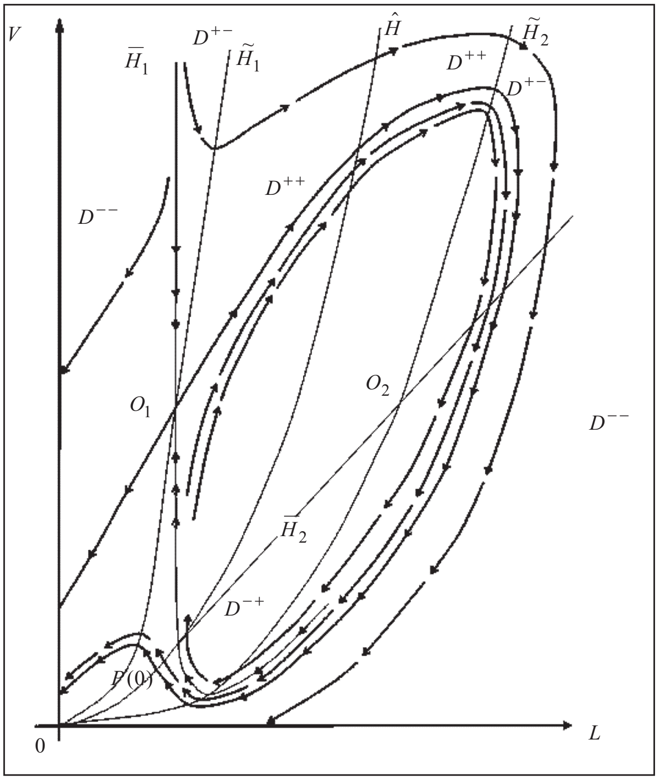

Fig. 5. Phase portrait in the case where the separatrix does not belong to the domain of attraction of points O_1 and O_2 for $\delta = 0$, where $\to\to\to$ are phase trajectories, $\to\!\!\to\!\!\to$ are separatrices of the point O_1.

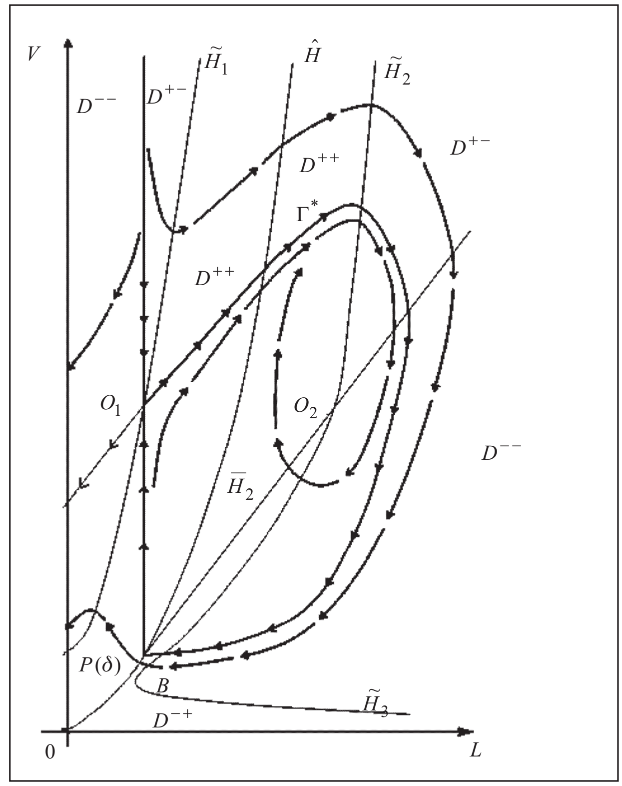

Fig. 6. Formation of unstable homoclinic curve at $\delta = \delta^*$, where $\to\to\to$ are phase trajectories, $\to\!\!\to\!\!\to$ are separatrices of the point O_1, $\longrightarrow$ is homoclinic curve.

We will finish the bifurcation analysis with the case where points of the separatrix emanating from O_1 belong to the influence domain of the stable focal point O_2 for $\delta = 0$. As δ increases, the point O_1 shifts to the left, and O_2 to the right, its influence domain expands. The phase trajectories that finish at O_2 have the form of contracting spirals. An increase in δ will not qualitatively change the phase portrait of the system. The available manpower L and the equilibrium demand V will vary in mutual proportion. Increase in the minimum wage allows involving additional manpower by intensifying immigration; however, it will scarcely affect the average income of employees.

Note also that the increase in the amplitude of economic cycles, typical of the case where O_2 is unstable focal point, can cause a transition from cyclic development to long-term economic recession if the phase point crosses the vertical straight line $L = \gamma^{-1/2}(\overline{W} - \delta)^{1/2}$. This scenario was observed in the numerical experiments.

Thus, the dynamics of the system with open monopsonic labor market is much like that with closed market, analyzed in [4]. Stable and unstable limit cycles may also be generated, as well as the trajectories of the form of contracting and expanding spirals. In the both cases, economic cycles are similar to the classical business cycle with the stages of growth, boom, recession, depression, and revival of economy. In contrast to a closed-market system, the transition from boom to recession and from depression to revival of economy for an open market may not be accompanied by a significant variation in prices. Devaluation of manpower because of intensive immigration during the boom and increase in the remuneration of labor due to the reduction in manpower at the final stage of depression play the same role as inflationary depreciation of savings and deflationary growth of actives in a system with closed labor market.

A peculiarity of a system with open labor market is that catastrophic recession may occur in it, accompanied by a loss of manpower because of emigration. The transition from cyclic development to this scenario is possible in case of increased amplitude of cycles. The growth of minimum wage is usually a stabilizing factor that prevents this transition.

3. SOME ASPECTS OF INTERACTION OF THE ECONOMIC SYSTEM WITH FOREIGN LABOR MARKET

In considering the model analyzed in the previous sections, some assumptions did not always fully represent the real economic situation. In particular, it was assumed that the number of potential employees (L) can drop to zero because of labor emigration for a sufficiently large difference in the remuneration of labor in the foreign and domestic markets. Note that there are categories of persons who are prone to migration less than others for any value of this difference. They include workers of preretirement age, who have a low motivation for the transition to a new place; persons with the underestimated capabilities and those who consider that their limited abilities (ignorance of language, impossibility to adapt to new conditions of life, etc.) will interfere achieving a success on a new place; persons who have obligations that complicate emigration, etc. [10]. Therefore, it is quite logical to assume that L cannot be less than some $L_{\min} > 0$, and the phase space of the system (7)–(9) is the set $E^{(1)} = \{(V, L): 0 \le V \le V_{\max},\ \ L_{\min} \le L \le L_{\max}\}$.

In the model considered above, it was implicitly assumed (through $L_{\max}$) that migration flows were upper bounded because of finite resources of the foreign market. We considered the case where $L_{\max}$ is so large that all singular points of the equations forming the model belong to its phase space. Of interest is the case where these assumptions are not valid.

Analyzing the system (7)–(9), we assumed $\overline{W}$ constant. Of interest is to consider it as a parameter varying like the parameter δ (see Sec. 2). Let us analyze how the behavior of the process being modeled varies in this case.

First, let us estimate the influence of the variation in $\overline{W}$ on the behavior of system (7)–(9) and the influence of the system dynamics on possible changes in $\overline{W}$ for different types of organization of the foreign labor market. Note than the increase in $\overline{W}$ shifts the stationary point O_1 of the system (7)–(9) to the right, and the stationary point O_2 to the left, i.e., the consequences of such an increase are opposite to the consequences of the increase in δ. If δ is sufficiently small, $\operatorname{tr}(A_2)$ decreases as $\overline{W}$ increases. Hence, the point O_2, being an unstable focal point for small $\overline{W}$, becomes a stable focal point of the system as this parameter increases. The domain $\{L: 0 \le L \le \gamma^{-1/2} (\overline{W} - \delta)^{1/2}\}$ of long-term recession resulted in collapse of the economy will expand. Thus, an increase in the remuneration of labor in foreign markets will change the behavior of cyclic processes, which become contracting spirals that end at the stationary point O_2 if the manpower L of the system is sufficiently large, and will lead to irreversible drop in production accompanied by intensified labor emigration if the manpower is low. A reduction in the remuneration of labor in foreign markets will produce the opposite effect.

Note that in contrast to a closed monopsonic market, where the remuneration of labor is usually smaller than that in a competitive market being in a similar situation [11], wages in an open market under conditions of long-run growth (or at the stage of growth in cyclically developed economy) are larger than the remuneration in the foreign market. If the latter is competitive and migration flows are comparatively small, the supply of manpower in this market will not profoundly change, and $\overline{W}$ will remain constant. Under such conditions during a long period, wages in the domestic labor market will be larger than in the foreign one. This, in particular, can explain the negative norm of monopsonic exploitation of the personnel of universities in largest research centers in the USA such as the Research Triangle Park in the North Carolina and the Boston agglomeration in the Massachusetts. Beginning in the 1960s, organizations located in these regions follow a coordinated employment and remuneration policy [6]; therefore, the corresponding segments of labor market are close to an open monopsonic market. Nevertheless, remuneration of labor in these segments is 10 to 15% greater than that in nearest regions, especially for persons who change the place of employment [12].

Constant inflow of manpower under the conditions of growing demand for the product manufactured is the main source of incomes of the monopolist employer increasing the output. This makes the employer to establish remuneration of labor greater than its equilibrium value for the foreign competitive market.

When the outflow of manpower from the foreign competitive market becomes significant, the labor supply in it decreases and $\overline{W}$ increases. Therefore, the economic dynamics changes. If the line $\overline{H}_1$ shifts to the right substantially, the growth or cyclic development of the economy is replaced by long-term recession. Since the type of the stationary point O_2 is changed and the domain of its attraction is extended, the phase trajectories that had earlier the form of monotonically increasing or closed curves passing in the vicinity of the limit cycle somewhat removed from this point will take the form of contracting spirals; the motion along these spirals will quickly finish near O_2.

Recession in the system being modeled generates an outflow of manpower to the foreign competitive market. The line $\overline{H}_1$ shifts to the left, and the stability and the domain of attraction of the point O_2 decrease, which may cause the revival of the economy and renewal of growth. Thus, cycles may occur in a system with open monopsonic labor market, generated by its interaction with the foreign market. They require a separate study, including that with the use of economic-mathematical modeling.

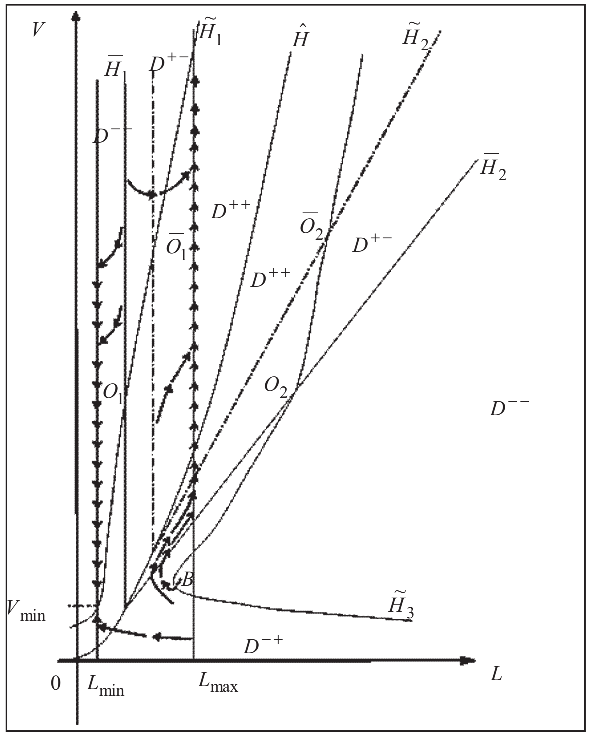

Fig. 7. Phase trajectories when the foreign and domestic markets interact, $\to\to\to$ are phase trajectories, $\twoheadrightarrow\twoheadrightarrow$ is sliding motion, $-\cdot-\cdot-$ is new position of $\overline{H}_1$ and $\overline{H}_2$ after increase in $\overline{W}$.

Let us consider the case of open monopsonic foreign market and cyclic development. In the interaction of the domestic and foreign markets, the coincidence or mismatch of cycle stages in these markets play an important role. The market being at the stage of growth obtains an additional short-term advantage from the recession in the market cooperating with it since the increased inflow of manpower provides the same output growth rates for smaller remuneration growth rates. At the same time, the growth of remuneration in the foreign market may aggravate the recession in the domestic market and make it irreversible if the phase point appears to the left of the line $\overline{H}_1$. Reducing the labor emigration can avoid this. Thus, it is expedient in a system with open monopsonic labor market to stimulate migration processes at the stage of their growth and to restrict them during the recession.

The coincidence of the stages of recession in the foreign and domestic markets may cause accelerated transition to growth. The coincidence of the stages of growth aggravates a competitive struggle for manpower.

Consider now the peculiarities of the dynamics of a system with the phase space $E^{(1)}$ for $L_{\min} > 0$. Similarly to the previous section, we assume that once the phase point reaches the boundary of the phase space, it continues sliding influenced by the vector field. Let $L_{\min} < \gamma^{-1/2}(\overline{W} - \delta)^{1/2}$ and the line $L = L_{\min}$ intersect the domains D^{--} and D^{-+} (Fig. 7). Then once the phase point reaches the boundary of the set $E^{(1)}$ in the domain D^{-+}, it continues sliding upwards along the line $L = L_{\min}$. If the phase point achieves the boundary of $E^{(1)}$ in the domain D^{--}, it will slide downwards. In both cases, the motion will obviously end at a singular point $(V_{\min}, L_{\min})$ of the system, which is the point of intersection of the lines $\widetilde{H}_1$ and $L = L_{\min}$. Collapse in the economy will be replaced with stagnation at the level that depends on the number of employees not prone to labor migrations. As this number increases, the output level $V_{\min}$ at which the recession stops grows; however, the output will not be recommenced. The point $(V_{\min}, L_{\min})$ will be a point of stable equilibrium of the economic system being modeled: for a small increase in V and (or) L, the recommenced motion will end at the same point.

Let us now consider how the line $L = L_{\max}$ passing to the left from the point O_2 influences the phase trajectories. Once the phase point crosses the straight line $L = L_{\max}$, it continues sliding along this straight line. Note that such a crosspoint may belong to the domain either D^{+-} or D^{++} (see Fig. 7). In the former case, the sliding is accompanied by a decreased demand V for the product manufactured by the employer; in the latter case, this demand increases. In both cases, the sliding finishes at the point of intersection of the straight lines $\widetilde{H}_1$ and $L = L_{\max}$. This point, not being a stationary point of the system (7)–(9), is a state of stable economic equilibrium. Small variations in V and L (the latter only decreases with $L_{\max}$ constant) will not bring the phase point beyond the domains D^{+-} and D^{++}; therefore, the recommenced motion will finish at the same equilibrium point. The upper bound for L does not influence the phase trajectories passing through the domains D^{--} and D^{-+} to the left from the straight line $L = L_{\max}$ since the motion of the phase point within these domains is accompanied by the decrease in L.

Thus, if migration flows are bounded from above, there may be either long-term recession or stabilization of the economy at the point of stable equilibrium, for which all the available manpower will be involved in the domestic labor market. Long-term recession may end in either stagnation (if there is a non-migrating part of able-bodied population) or collapse (if it is absent).

The theoretical analysis was implemented with the numerical experiments conducted by V. Barinov, a student of the Taurida National University. Let us consider some of these experiments.

4. RESULTS OF NUMERICAL EXPERIMENTS

The experiments consist in solving the system of differential equations (7)–(9) by numerical methods for different values of its parameters and initial approximations. The purpose of the experiments, along with illustration of the results of the theoretical analysis of the above-mentioned system, is to analyze the features of the dynamics of the economic processes being modeled that were not revealed in the theoretical analysis. Since the conjuncture of an open labor market substantially depends on the initial value of manpower, the computations were carried out for a series of initial approximations that differ only in $L(0)$. Unfortunately, because of the limited space, we cannot present the results of all such computations; therefore, we will dwell on the most typical of them. The other results do not contradict the data presented below.

Figure 8 plots the trajectories on the plane $V0L$ obtained as solutions of the system (7)–(9) for $a = 4.5$, $\overline{W} = 0.5$, $\gamma = 0.19$, and different δ for ten initial approximations, with $V(0) = 5$ and $L(0)$ taking successively the values of 1.6, 1.7, 1.8, 2.5, 3.0, 4.0, 4.5, 5.0, 5.3, and 5.5.

The results of the solution of the system for these initial approximations for $\delta = 0$ are presented in Fig 8a. All the initial approximations, except for the point $V(0) = 5$, $L(0) = 1.6$ are located to the left from the imaginary line $L = \gamma^{-1/2}(\overline{W} - \delta)^{1/2} = 1.622$ (all the computations whose results are presented below are accurate to within the third decimal place). The corresponding phase trajectories begin on the sets D^{+-}, D^{++}, and D^{--}. The stationary point O_2 of the system is an unstable focal point since $\operatorname{tr}(A_2) = 0.686 > 0$. Hence, even the phase trajectories that pass relatively close to O_2 are not attracted by this point, and there is no cyclicity in the system being modeled. The end of all the phase trajectories is associated with collapse of the economy, where either $V = 0$ or $L = 0$.

The solution results for a similar system for $\delta = 0.1$ are presented in Fig. 8b. All the initial approximations are now to the right from the line $L = \gamma^{-1/2}(\overline{W} - \delta)^{1/2} = 1.451$. The stationary point O_2 remains to be unstable since $\operatorname{tr}(A_2) = 0.375$. For δ close to 0.1, a homoclinic loop originates, which is stable since the saddle value of the point O_1 is $S_1 = 2\gamma^{1/2}(\overline{W} - \delta)^{1/2} - 1 = -0.499 < 0$. The process of its origination is extremely unstable; therefore, it is as a rule impossible to trace it during numerical experiments. However, the origination of a limit cycle close to a homoclinic loop is plainly seen in Fig. 8b.

Figures 8c and d show how the solutions of the system vary with further increase in δ. For $\delta = 0.2$, there is a stable limit cycle that contracts into the point O_2 as δ increases. The point becomes stable, and the phase trajectories have the form of rapidly contracting spirals (Fig. 8d). Thus, the process being modeled passes (accurate to computation errors in model computations) into the equilibrium state after several sequential stages of one economic cycle but without completing this cycle. Note that such dynamics was also observed for the models of economic systems with closed monopsonic labor market considered in [4].

Figure 9 presents the results of the next series of numerical experiments for the system (7)–(10) with the parameters $a = 4$, $\overline{W} = 0.9$, $\gamma = 0.5$ and for the initial approximations $V(0) = 2$ and $L(0)$ taking successively the values of 1.0, 1.2, 1.35, 1.4, 1.5, 1.9, 2.1, 2.3, and 2.5.

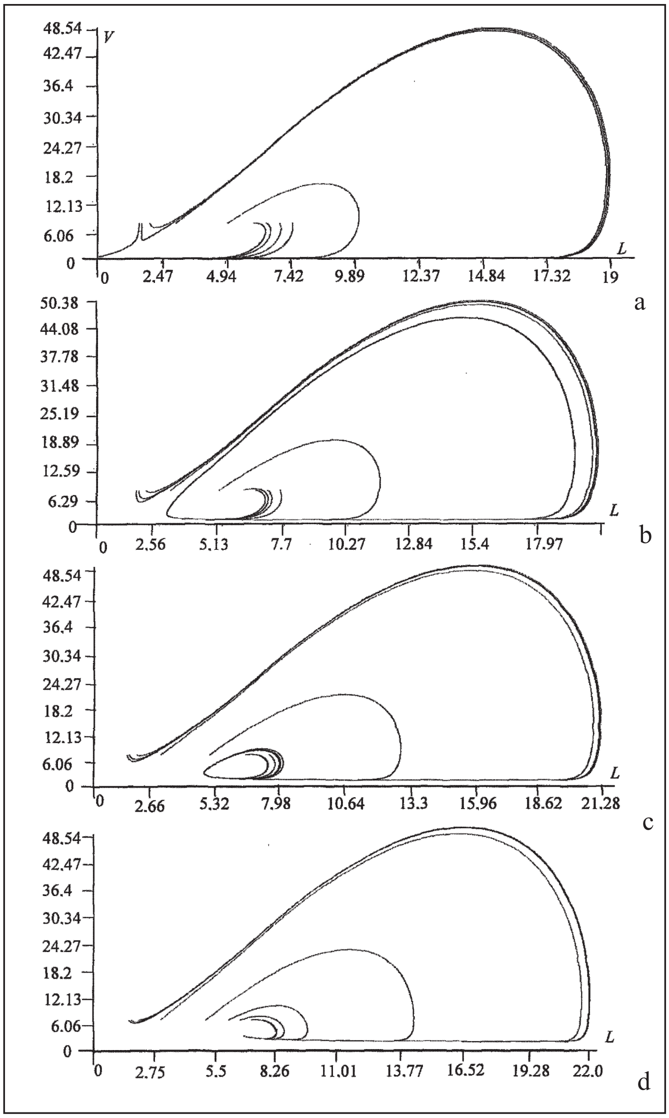

Fig. 8. Trajectory for the first series of initial approximations: $\delta = 0$ (a); $\delta = 0.1$ (b); $\delta = 0.2$ (c); and $\delta = 0.4$ (d).

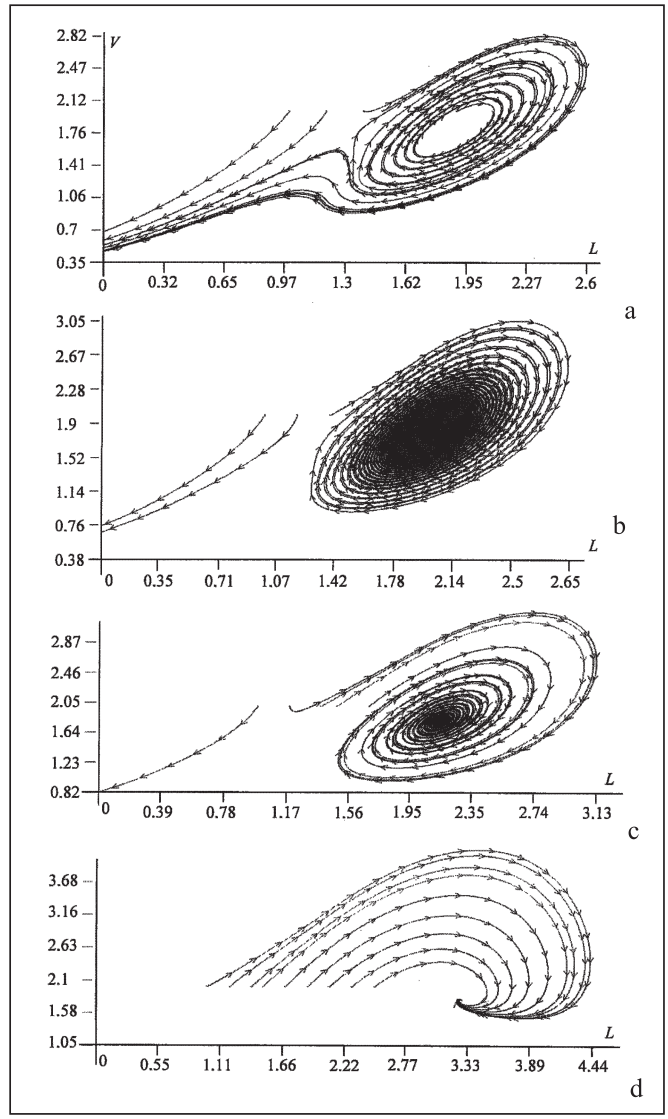

Fig. 9. Trajectory for the second series of initial approximations: $\delta = 0$ (a); $\delta = 0.1$ (b); $\delta = 0.3$ (c); and $\delta = 0.6$ (d).

Figure 9a shows the phase trajectories for $\delta = 0$. The initial points $(2; 1)$ and $(2; 1.2)$ are located to the left from the vertical straight line $L = \gamma^{-1/2}(\overline{W} - \delta)^{1/2} = 1.342$; therefore, the phase trajectories that begin at these points do not abandon the set D^{--} and end with a collapse for $L = 0$. Cyclic dynamics is observed for other initial approximations. Since O_2 is an unstable focal point, these trajectories take the form of expanding spirals. They correspond to economic cycles with increasing amplitude but without a tendency to long-term growth. For a sufficiently large amplitude of the cycle, the phase point intersects the above-mentioned vertical straight line, and cyclic development is replaced with long-term recession finished for $L = 0$. The point O_2 becomes stable as δ grows. Figure 9b demonstrates the merging of the stable and unstable limit cycles for $\delta = 0.1$. As δ increases ($\delta = 0.3$), the system has a stable focal point with a wide attraction domain and a saddle point (Fig. 9c). The phase trajectories, except for the straight line $L = 1.095$ beginning at the point $(2; 1)$ and located to the left from the boundary of the domain D^{--} have the form of contracting spirals that finish at the stable focal point.

For large δ ($\delta = 0.6$), the phase trajectories correspond to the fast transition of the system to the equilibrium state (Fig. 9d). Such a transition can take only several stages of one cycle.

Thus, a sufficiently high minimum wage under conditions of open monopsonic market described by the model (7)–(10) can substantially influence the economic dynamics by preventing a catastrophic recession for small V and L and by changing the form of the cycles. The cycles that are closer to the classical type for small δ take the features of postclassical cycles for large values of this parameter.

5. CONCLUSIONS AND LINES OF FURTHER ANALYSIS

Analyzing the economy with a monopsonic labor market, we draw the following conclusions.

1. Three types of dynamics are possible in the economic system under study: long-term growth, recession, and cyclic development. The first type requires additional manpower to be involved and a higher remuneration of labor to be kept as compared with the foreign market. In case of limited manpower because of stimulated immigration, the development of the system will be completed at the point of its stable equilibrium with the position determined by the value of the above-mentioned manpower.

2. Long-term recession begins if manpower of the system decreases due to emigration below a critical level. This level depends on labor productivity, additional expenses of the employer for the purchase of manpower, and on the remuneration of labor in the foreign market. The recession is accompanied by labor emigration and decrease in the remuneration in the domestic (open monopsonic) market. With the assumption that emigration of all potential employees is possible in case of a sufficiently large difference in the remuneration in the foreign and domestic markets, the recession will end in collapse of the economy because of the total loss of manpower. Otherwise, the system will reach stable equilibrium, which corresponds to the stagnation at a low level of production. The position of this state depends on the number of potential employees who do not migrate at any remuneration ratio in the foreign and domestic markets.

3. The cyclic development in a system with open monopsonic labor market is similar in many respects to the dynamics of a system with closed market considered in [4]. In both cases, cycles similar to a classical business cycle are formed, with the stages of growth, boom, recession, stagnation, and revival of the economy. In the models of both above-mentioned systems, homoclinic loops and limit cycles (stable and unstable) may occur, the trajectories in the form of contracting and expanding spirals may originate, and the system may pass to the equilibrium state after successively passing several stages of one cycle. The cycle-formation mechanisms in a system with open labor market have special features. The inflow of manpower due to the intensified labor immigration during the boom increases the labor supply, which reduces the remuneration of labor. This accelerates and aggravates the recession. The outflow of manpower during the depression reduces the supply of manpower. As a result, real wages can grow under certain conditions, which revives the economy. Thus, the role of migration processes in cycle formation in the economy with open labor market is similar to the inflationary depreciation of savings and deflationary growth of actives in case of closed market.

4. Introducing minimum wage stabilizes the system. The possibility of catastrophic reduction in demand is excluded; trajectories with constant or increasing amplitude of the cycles get the form of contracting spirals. For sufficiently large minimum wage, the equilibrium state of the system to which such trajectories converge becomes a stable focal point with a wide attraction domain, i.e., provides fast stabilization of the process with significant scatter of its initial states.

5. The interaction of the domestic and foreign open monopsonic labor markets may adversely affect the dynamics of the processes if the stages of growth and (or) revival of the economy in these markets coincide. At the same time, the coincidence of the recession stages will accelerate the revival of the economy. The interaction of the markets at their transition to stable equilibrium can equalize the remuneration in these markets.

In our opinion, the main lines of further analysis of the processes of interaction of labor markets with imperfect competition may be as follows:

(a) analysis of alternative behavior models of a monopolist employer, for example, considering the case where the employer takes into account the influence of the remuneration of labor on the demand for the production;

(b) studying the models of economy with other than monopsonic types of imperfect competition in open labor market, for example, a bilateral monopolistic competition;

(c) accounting for asymmetric information and different reactions to the variation in the remuneration of labor of migrants and the non-migrating part of able-bodied population.

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